

## Significant Figures in Measurement and Calculations Reader

It is important to always label your numbers with units because in Chemistry class our numbers represent measurements. It is also important to only report digits that are reliable – we call these *significant figures*. If we report more digits than we can adequately ensure are reliable then our results will have too high of a margin of error. When we report a number we report:

*Figures (digits) definitely known + One estimated figure (digit)*

In class you will hear this expressed as "all of the digits known for certain plus one that is a guess."

### Recording Measurements

When you read an instrument (ruler, thermometer, etc), you report the reading with the right number of "sig figs" so your data is reliable. For example, in the image below, note the reading marked A. This reading is definitely beyond the 7 cm mark and also beyond the 0.8 cm mark. We know the 7.8 with certainty. We then *estimate* that the reading is between the 7.8 mark and the 7.9 mark. So, we estimate the length as 0.05 cm more than 7.8 cm. All of these have meaning and are therefore significant. We express the reading as 7.85 cm, accurate to three sig figs. All of these figures, 7.85, can be used in calculations. In reading B we see that 9.2 cm is definitely known. We can use one estimated digit in our reading, so we estimate the next digit to be zero. Our reading is reported as 9.20 cm. It is accurate to three sig figs.



### Rules for Zeros

If a zero represents a measured quantity, it is a significant figure. If it merely locates the decimal point, it is not a sig fig.

- **Zero Inside a Number.** 9.04 cm, the zero represents a measured quantity, just as 9 and 4, and is, therefore, a significant number. A zero trapped or "captive" between any of the other digits is a significant figure.
- **Zero at the Front of a Number.** 0.46 cm, the zero does not represent a measured quantity, it just locates the decimal point. It is not a sig fig. Also, in the measurement 0.07 kg, the zeros are used to locate the decimal point and are, therefore, not significant. Zeros at the front of a number are "leading zeros" and are not significant figures.
- **Zero at the End of a Number AFTER a Decimal Point.** 11.30 cm, the zero is an estimate and represents a measured quantity. It is significant. Another way to look at this: The zero is not needed as a placeholder, and yet it was included by the person recording the measurement. It must have been recorded as a part of the measurement, making it significant. Zeros to the right of the decimal point, and at the end of the number, are significant figures.
- **Zeros at the End of a Number with NO Decimal Point.** Zeros at the end of a number when there is no decimal point are functioning as place holders and are not actual measured digits. If a distance is reported as 1600 feet, one assumes two sig figs. It could have been 1604 feet, or 1683 feet, we don't know. We only know the number is reliable to the hundreds place so we only report 1600.
- One trick – putting a number in scientific notation can help reduce the number of non significant zeros you are writing down.

$1.6 \times 10^3$ feet	Two significant figures
$1.60 \times 10^3$ feet	Three significant figures
$1.600 \times 10^3$ feet	Four significant figures

**Sample Problem #1:** Underline the significant figures in the following numbers.

(a) 0.0420 cm	answer = 0.04 <u>20</u> cm	(e) 2403 ft.	answer = <u>2 403</u> ft.
(b) 5.320 in.	answer = <u>5.320</u> in.	(f) 80.5300 m	answer = <u>80.5300</u> m
(c) 10 lb.	answer = <u>10</u> lb.	(g) 200. g	answer = <u>200</u> g
(d) 0.020 ml	answer = 0.0 <u>20</u> ml	(h) $2.4 \times 10^3$ kg	answer = <u>2.4</u> $\times 10^3$ kg

### Rounding Off Numbers

In reporting a numerical answer, one needs to know how to "round off" a number to include the correct number of significant figures. Even in a series of operations leading to the final answer, one must "round off" numbers. There are different sets of rules out there, the rules here are common and well accepted rules:

1. Less than 5, round down
2. Greater than 5, round up
3. If 5 followed by any number other than 0 round up
4. If 5 followed only by zeros, and the previous number was odd, then round up
5. If 5 followed only by zeros, and the previous number was even then round down

**Sample Problem #2:** Round off the following to three significant figures.

(a) 3.478 m	answer = 3.48 m	(c) 5.333 g	answer = 5.33 g
(b) 4.8055 cm	answer = 4.81 cm	(d) 7.999 in.	answer = 8.00 in.

### Multiplication

In multiplying two numbers, you should inspect the numbers multiplied and find which has the least number of sig figs. This is the number of sig figs you should have in your answer (the product). Thus the answer to  $0.024 \times 1244$  would be rounded off to contain two sig figs since the factor with the lesser number of sig figs (0.024) has only *two* such figures.

**Sample Problem #3:** Find the area of a rectangle 2.1 cm x 3.24 cm. =  $6.804 \text{ cm}^2 \rightarrow 6.8 \text{ cm}^2$

2.1 contains two sig figs, while 3.24 contains three. Our product should contain no more than *two* sig figs. Therefore, round to  $6.8 \text{ cm}^2$

**Sample Problem #4:** Find the volume of a rectangular solid 10.2 cm x 8.24 cm x 1.8 cm =  $151.2864 \rightarrow 150 \text{ cm}^3$

The number with the fewest sig figs is 1.8 cm. It contains two sig figs so you round your final answer to 2 sig figs.

### Division

In dividing two numbers, the answer (quotient) should contain the same number of sig figs as are contained in the number (divisor or dividend) with the least number of sig figs. Thus the answer to  $528 \div 0.14$  would be rounded off to contain *two* sig figs. The answer to  $0.340 \div 3242$  would be rounded off to contain three significant figures.

**Sample Problem #5:** Calculate  $20.45 \div 2.4 = 8.52083 \rightarrow 8.5$

2.4 has fewer sig figs than the 20.45. It has only *two* sig figs. Therefore, our answer should have no more than two sig figs so round to 8.5.

## Addition and Subtraction

In adding (or subtracting), write down the numbers, being sure to keep the decimal places stacked under each other, and add (or subtract). Next, note which column contains the first estimated figure. This column determines the last decimal place of the answer. After the answer is obtained, it should be rounded off in this column. In other words, round to the least number of decimal places in you data.

**Sample Problem #6:** Add 42.56 g + 39.460 g + 4.1g

*Solution:*

$$\begin{array}{r} 42.56 \text{ g} \\ 39.460 \text{ g} \\ + 4.1 \text{ g} \\ \hline \text{Sum} = 86.120 \text{ g} \end{array}$$

Since the number 4.1 only extends to the first decimal place, the answer must be rounded to the first decimal place, yielding the answer 86.1 g

## Average Readings

The average of a number of successive readings will have the same number of decimal places that are in their sum.

**Sample Problem #7:** A graduated cylinder was weighed three times and the weightings were 12.523 g, 12.497 g, 12.515 g. Calculate the average weight. *Solution:*

$$\begin{array}{r} 12.523 \text{ g} \\ 12.497 \text{ g} \\ + 12.515 \text{ g} \\ \hline \text{Sum} = 37.535 \text{ g} \end{array}$$

In order to find the average, the sum is divided by 3 to give an answer of 12.51167. Since each number extends to three decimal places, the final answer is rounded to three decimal places, *yielding a final answer of 12.512 g.* Notice that the divisor of 3 does not effect the rounding of the final answer. This is because 3 is an exact number - known to an infinite number of decimal places.

## Exact Numbers

Exact numbers have infinite number of sig figs. If I were to count how many students were in my classroom I would know there were exactly 35. It isn't possible to have 35.4 students in the room, or 35.8 students. I have *exactly* 35 students. I could report it with an infinite number of zeros but that would clearly not be practical. When performing calculations involving an exact number, you assume the number is infinite sig figs and therefore doesn't determine how many sig figs are in your final answer. Be careful though - a "whole number" isn't always an exact number. If I told you there were four people in my family that has infinite sig figs. But if I told you that I walked 4 miles that is a measurement and only has 1 sig fig. I could have walked 3.7 miles, or 4.2 miles etc.

**Sample Problem #8:** Convert 12 minutes into hours =  $12 / 60 = 0.20 \text{ hours}$  → there are exactly 60 minutes in an hour so the 60 is an exact number. It will not limit your sig figs even though it looks like it only has 1 sig fig. It actually has infinite. Therefore your answer can be reported to two sig figs.

## Logarithms, and values like pH and pKa

In answers to logarithmic calculations (like pH and pKa values), only those numbers to the right of the decimal place count as significant. These digits are called the "mantissa."

**Sample Problem #8:** pH = 10.26 has only two significant figures (this represents a  $[H^+] = 5.5 \times 10^{-11} \text{ M}$ )  
pKa = 4.730 has three significant figures (this represents a  $K_a = 1.86 \times 10^{-5}$ )

When you take the log of a number with X number of significant figures, the result should have X number of decimal places. The number in front of the decimal place only indicates the order of magnitude, it isn't a significant figure.

**Sample Problem #9:**  $\log(2.4 \times 10^3) = 3.3802 \rightarrow 3.38$ , there were two sig figs in the number you took the log of. So your answer should have two sig figs. 3.38 is two sig figs because the answer to a logarithm problem only counts the numbers after the decimal as significant. Another example:  $\log(5.5 \times 10^{-11}) = 10.2596 \rightarrow 10.26$ ,  $5.5 \times 10^{-11}$  only had two sig figs, so the final answer should only have two sig figs. Since the answer came from a logarithm only the numbers after the decimal are significant so 10.26 only has two sig figs.

## Multiple Operations

When performing multiple operations you need to take into account the "order of operations" Remember the mnemonic:

*Please Excuse My Dear Aunt Sally* = Parenthesis, Exponents, Multiplication, Division, Addition, Subtraction

**Sample Problem #10**  $2.0000(1.008 \text{ g}) + 15.99 \text{ g} = 18.01$

Perform the multiplication first →  $2.0000(1.008 \text{ g}) = 2.016 \text{ g}$ , 4 sig figs because 1.008 was the smallest number of sig figs.

Then, perform the addition →  $2.016 + 15.99 = 18.006 \rightarrow$  round to 2 decimal places since 15.99 only has 2 decimal places.

*These last two categories are a bit more complicated to explain in words, so here are some videos that you might find helpful:*



<https://tinyurl.com/3he62jz8>

## Scientific Notation

*(Note – the video I am putting here shows doing these by hand. We will typically have a calculator which helps! But you need to still report your answer with the right sig figs!)* When multiplying or dividing scientific notation numbers, the sig figs are determined by the number with the least amount of sig figs. Make sure you are careful that you check what your exponent ends up being, even if both numbers have the same exponent to start doesn't mean it won't change when you have your final answer! Common mistake!

**Sample Problem #11**  $(2.0 \times 10^{12}) / (8.330 \times 10^9) = 2.40096 \times 10^3 \rightarrow 2.4 \times 10^3$ , only 2 sig figs because  $2.0 \times 10^{12}$  only had 2 sig figs.



<https://tinyurl.com/48jjyeac>

When adding or subtracting scientific notation numbers, you have to FIRST have the same exponent for each number before you can determine the true number of sig figs. If your numbers have the same exponent then just count the number of decimal places in the mantissa and that will be the number of decimal places to use in your final answer. You make sure your answer is using the same exponent that you started the problem with, adjust your sig figs and THEN you reformat your answer if needed to be a more proper scientific notation format.

**Sample Problem #12**  $(2.113 \times 10^4) + (9.2 \times 10^4) = 11.313 \times 10^4 \rightarrow 11.3 \times 10^4 \rightarrow 1.13 \times 10^5$  \*Calculators can cause real problems for these, be careful! Only 1 decimal place because  $9.2 \times 10^4$  only had one decimal place so it limits your answer to  $11.3 \times 10^4$ . But that is not good scientific notation form so adjust it to have one number and then the decimal which results in  $1.13 \times 10^5$ .

If your numbers do not have the same exponent, convert one of them so it matches the other. It won't be in perfect scientific notation form with one number and then the decimal but it doesn't matter because you are just trying to determine the sig figs. Once your numbers both have the same exponent determine which has the fewest decimal places in the mantissa and that will be the number of decimal places to use in your final answer.

**Sample Problem #13**  $(1.032 \times 10^4) + (2.672 \times 10^5) \rightarrow$  convert so they have the same exponent even if it makes your number look weird format.  $(1.032 \times 10^4) + (26.72 \times 10^4) \rightarrow$  Now follow normal adding subtracting rules →  $27.752 \times 10^4 \rightarrow$  round to two decimal places because  $26.72 \times 10^4$  only had two decimal places so that limits your answer →  $27.75 \times 10^4 \rightarrow$  now fix your format to put it back in proper scientific notation format → Final answer of  $2.775 \times 10^5$