Significant Digits

Science is all about observing and measuring things. In every measurement we make, there is some error involved. You might examine a piece of copper wire and say that it appears to be "about 5 cm long". If you want a more accurate measurement, you grab a cheap ruler and report that its length is actually 4.7 cm long. But if you need even more accuracy, you get a high-quality ruler with more graduations on it and report that the length is really 4.73 cm long.

You made 3 measurements:

Rough estimation:	5 cm
Quick ruler measurement:	4.7 cm
Most careful measurement:	4.73 cm

Notice that the more accurate the measurement, the more digits there are. **When we make measurements, we record all the digits that are certain, plus one digit that is estimated.** These digits are called **significant digits** – they are meaningful.

When you eyeball the copper wire to report its length, you wouldn't say that it appears to be 4.73 cm long. No one would believe you! By saying just "5 cm", you're saying that you really aren't <u>certain</u> about any digits … but you're going to <u>estimate</u> one digit and you report "5 cm".

Using the ruler … it becomes clear that the length is not really 5 cm … it's between 4 and 5 cm. You look at the scale on the ruler and estimate one more digit … and report "4.7 cm".

Finally, with the best ruler, you can see it's length is longer than 4.7 but less than 4.8 cm. So you estimate one more digit and report the length as "4.73 cm".

Counting Significant Digits

Follow these rules to count the number of significant digits in a recorded measurement:

1. All **non-zero digits** in the measurement are significant.
2. **Leading zeros** are never significant. For example, in the measurement "0.0002 g", none of the zeros are considered significant because they are all "leading zeros".
3. **Zeros in between non-zero digits** are always significant. So in the measurement "0.002001 s", there are 4 significant digits. The first 3 zeros don't count … but all the other digits do.
4. **Trailing zeros** can be tricky.
    a. If a measurement **includes a decimal point**, they are significant. For example, "0.0200 g" has 3 significant digits. The two leading zeros don't count … but the two trailing zeros do because they come after a decimal point. In the volume "20. L" there would be two significant digits. The trailing zero counts because a decimal point is visible.
    b. If the measurement **does not include a decimal point**, it's unclear whether the trailing zeros are significant. Some books will say they are not significant. For example, most books would say the volume "5000 L" has only one significant digit – none of the trailing zeros would count.

For measurements involving **scientific notation**, only the number in front of the "power of 10" is examined for significant digits. So the measurement 7.50×10^3 kg has <u>three</u> significant digits (7.50).

For each measurement below, count and record the number of significant digits:

	Measurement	# of Significant Digits		Measurement	# of Significant Digits
1.	12.25 mL	4	11.	180. mL	3
2.	0.017 g	2	12.	22.4 L	3
3.	0.000100 cm	3	13.	200 g	1
4.	2.90×10^{-3} mL	3	14.	6.020×10^{15} s	4
5.	0.005010 kg	4	15.	5.000 kg	4
6.	7500 s	2	16.	5 kg	1
7.	8.0×10^4 m	2	17.	5020 kJ	3
8.	0.1500 g	4	18.	0.090100 g	5
9.	4000. J	4	19.	7500. s	4
10.	24 700 mm	3	20.	0.400 mol	3

Throughout the course, you'll do calculations and then have to round off the answer displayed on your calculator so that it has a certain number of significant digits.

For example, suppose you do a calculation to find the area of a circle. Your calculator displays the area as 74.80250293 cm^2.

Required # Significant Digits	**Round off and record the area as …**
1	70 cm^2 or 7×10^1 cm^2
2	75 cm^2 or 7.5×10^1 cm^2
3	74.8 cm^2 or 7.48×10^1 cm^2
4	74.80 cm^2 or 7.480×10^1 cm^2

Try that here. Be careful … some answers **MUST** be written in **scientific notation**.

	Calculator Display	# of Significant Digits Needed	Rounded Off Answer
1.	12.1470091	3	12.1
2.	200.4709023	3	200.
3.	17300.279	4	$1.730E^{4}$
4.	2.07091147	3	2.07
5.	60.0765523	1	$6E^{1}$
6.	0.0002703462	2	0.00027
7.	$2.9020023 \times 10^{-12}$	1	$3E^{-12}$
8.	$5.002452937 \times 10^{26}$	3	$5.00E^{26}$
9.	0.0000027051	3	$2.70E^{-6}$
10.	0.000060709	2	$6.1E^{-5}$
11.	$1.570927333 \times 10^{12}$	1	$2E^{12}$
12.	$4.230176947 \times 10^{-15}$	2	$4.2E^{-15}$
13.	0.17073092754	3	0.171
14.	0.17073092754	2	0.17
15.	0.17073092754	1	0.2
16.	15007621.35	1	$2E^{7}$
17.	15007621.35	2	$1.5E^{7}$
18.	15007621.35	3	$1.50E^{7}$
19.	6	2	6.0
20.	72	4	72.00

Working with Significant Digits

In chemistry you'll often need to perform simple arithmetic operations with measurements. You may need to add/subtract or multiply/divide measurements, for example. When doing so, follow the rules here to understand how to round off your answers.

Adding & Subtracting

When you add or subtract with measurements, look at the number of decimal places in each measurement. **Round off the answer so that it has the same number of decimal places as the measurement that had fewest!** In other words, keep the smallest number of **decimal places** in your answer.

e.g. Add:	22.57 cm	+	1.029 cm	+	11.1 cm	=	34.699 cm	=	**34.7 cm**
	2 decimal places		*3 decimal places*		*1 decimal place*		*Calculator's answer*		*Correctly rounded to 1 decimal place*

e.g. Add:	22 min	+	1.02 min	+	3.757 min	=	26.777 min	=	**27 min**
	0 decimal places		*2 decimal places*		*3 decimal place*		*Calculator's answer*		*Correctly rounded to 0 decimal places*

e.g. Add:	5.670 m	+	4.33 m	=	10 m	=	**10.00 m**
	3 decimal places		*2 decimal places*		*Calculator's answer*		*Correctly rounded to 2 decimal places*

Data Table:

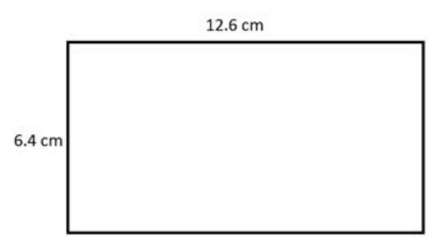
Mass of dish with liquid before heating:	22.570 g	
Mass of dry dish after heating:	15.070 g	
Mass of liquid that evaporated:	**7.500 g**	*(the calculator display said 7.5)*

Multiplying and Dividing

When you multiply or divide with measurements, look at the number of significant digits in the numbers being multiplied and/or divided. **Round off so that your answer has the same number of significant digits as the measurement that had the fewest.** In other words, keep the smallest number of **significant digits** in your answer.

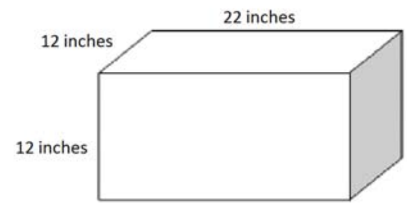
e.g. The **area** of this rectangle can be calculated:

$$Area = l\ x\ w = (6.4\ cm)(12.6\ cm) = 80.64 = 81\ cm^2$$



e.g. The **volume** of this solid can be calculated:

$$V = lwh = (12\ in)(12\ in)(22\ in) = 3168\ in^3 = 3.2x10^3\ in^3$$



e.g. A metal cube with edges 3.1 cm long has a mass of 42.718 g. Calculate the **density** of the cube.

First find the volume: $V = s^3 = (3.1\ cm)^3 = 29.79\ cm^3$

Now find density: $d = \frac{m}{V} = \frac{42.72\ g}{29.79\ cm^3} = 1.4\ g/cm^3$

Notice: The volume should have kept only 2 significant digits. Keeping one or two extra digits is smart if this is an intermediate answer and you're going to do a further calculation with it … but then you ***round off the final answer with the correct number of significant digits in mind****! This reduces "rounding error" in a final answer.*

Some numbers are considered "**pure numbers**" – they are not actually measurements. They are ignored when rounding off your final answers. For example, by definition, 1 in = 2.54 cm *exactly*.

e.g. Convert 6.3 in to "cm": $6.3\ in\ x\ \frac{2.54\ cm}{1\ in} = \mathbf{16\ cm}$

Keep 2 significant digits because of "6.3 in". The "2.54 cm" and "1 in" were exact numbers.

Calculations with Significant Digits

1) Complete the following addition & subtraction questions. Include appropriate units.

a) 12 cm + 3.1 cm + 79.71 cm = **94.8 cm**

b) 0.086 cm + 0.062 cm + 0.14 cm = **0.28 cm**

c) 2.419 g + 3.91 g + 7.0518 g = **13.38 g**

d) 30. g + 16.82 g + 41.07 g + 85.219 g = **173. g**

e) 13.0 cm + 89.25 cm + 6.00 cm = **108.3 cm**

f) 83.46 cm + 107.05 cm + 26.628 cm = **217.14 cm**

g) 0.0653 g + 0.07654 g + 0.0432 g = **0.185 g**

h) 3.25×10^{-4} cm + 4.6×10^{-5} cm = **$3.7E^{-4}$ cm**

i) 63.489 mL + 126.2 mL + 12.05 mL = **201.7 mL**

j) 2.3×10^2 g + 5.62×10^2 g + 3.852×10^2 g = **1177.2 g**

k) 41.025 cm – 23.28 cm = **17.75 cm**

l) 289 g – 43.7 g = **245.3 g**

m) 145.63 mL – 28.9 mL = **116.7 mL**

n) 62.47 g – 39.9 g = **22.6 g**

2) Complete the following multiplication & division questions. Include appropriate units.

a) 2.89 cm x 6.01 cm = **17.4 cm^2**

b) $\frac{172.931\ g}{8.5\ mL} =$ **20. g/mL**

c) 5.000 mm x 7.3216 mm = **36.61 mm^2**

d) $\frac{25.273\ km}{2.03\ h} =$ **12.4 Km/h**

e) 5 cm x 5 cm = **30 cm^2**

f) 5.0 cm x 5 cm = **30 cm^2**

g) 5.0 km x 5.0 km = **25 km^2**

h) 4.218 cm x 6.5 cm = **27 cm^2**

i) (4.8×10^2 m)(2.101×10^3 m) = **1.0E^6 m^2**

j) $\frac{2.5\ x\ 10^4\ m^3}{(2.13\ m)(6.74\ m)} =$ **1.7E^3**

k) (9.13×10^{-4} cm)(1.2×10^{-3} cm) = **1.1E^{-6} cm^2**

l) 5.08 m x 1.2000 m = **6.10 m^2**

m) $\frac{527.2\ cm^3}{4.1\ cm} =$ **1.3E^2 cm**

n) $\frac{(2.68\ x\ 10^2\ g - 2.53\ x\ 10^2\ g)}{17.25\ mL} =$ **0.8696 g/mL**

3) Complete the data table below to find the density of metal pellets.

	Trial 1	Trial 2	Trial 3
Mass of weighing boat, g	22.153	22.274	22.100
Mass of weighing boat with pellets, g	34.279	34.352	34.053
Volume water in cylinder, mL	62.4	51.9	58.7
Volume water with pellets, mL	71.0	60.2	67.6
Mass of Pellets, g	**12.126**	**12.078**	**11.953**
Volume of Pellets, mL	**8.60**	**8.30**	**8.90**
Density of Pellets, g/mL	**1.41**	**1.46**	**1.34**
Average Density, g/mL	**1.40**		

4) Use unit multipliers to convert these units. Express answers with appropriate significant digits.

a) 72.53 in to "cm"

72.53 in X 2.54cm / 1in = 184.2 cm

b) 17.1 h to "s"

17.1 h x 3600s / 1h = 6.16E^4 s

c) 5.73×10^4 g to "kg"

6.73E^4 g x 1kg / 1000g = 67.3 kg

d) 57.00 mL to "L"

57.00 mL x 1L / 1000 mL = 0.0570 L

e) 2.10 g to "kg"

2.10 g x 1kg / 1000 g = 0.00210 kg

f) 1.00 h to "min"

1.00 h x 60 min/ 1 h = 60.0 min

g) 5.1 cm to "in"

5.1 cm x 1 in/ 2.54 cm = 2.0 in

h) 4.3 ft to "cm"

4.3 ft x 12in/1ft x 2.54 cm/1in = 1.3E^2 cm

i) 16 y to "min"

16 y x 365d/y x 24h/d x 60min/h = 8.4E^6 min

j) 10.0 km to "cm"

10.0 km x 1000m/km x 100 cm/m = 1.00E^6 cm